Relativistic re-evaluation of the Michelson-Morley experiment

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by

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Abstract

The aim of the Michelson-Morley experiment was to assess the speed of earth moving through ether. A change in the interference patterns should have proven a reduction of the speed of light in the direction of earth's motion. The experiment failed. The idea of ether as a light medium was dropped and replaced by Einstein's special theory of relativity (STR).

On the occasion of the centennial of Einstein's *annus mirabilis* a project has been started where a virtual reality is programmed entirely on the basis of the mathematical laws of STR. To this end, a transformation matrix had to be created first, enabling to convert all various states of differently moving inertial systems into one another. It was likewise planned to reproduce the Michelson-Morley experiment within this virtual reality, and the result was utterly astonishing: When calculating the signal propagation time and the number of wave maxima - which are in fact responsible for interferences - it turned out that the speed of earth was canceled out and did not affect any changes in the interference patterns.

In other words, the Michelson-Morley experiment is no longer suitable to prove or disprove the existence of ether. However, this does not mean that STR is loosing its validity – quite the contrary! For this effect only arises if both, time dilatation and length contraction as per Einstein are considered in the calculation.

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Introduction

Up to the year 1887 it was a considered opinion of the scientific community that light waves propagate through a medium just like any other type of wave. The medium was assumed to be a mass-less stationary framework pervading the whole universe and was called ether. In 1887, Albert Abraham Michelson and Edward Morley tried to prove the existence of ether and wanted to measure the relative speed of earth moving through ether. They expected that in the direction of earth's motion the speed of light would be reduced by the speed of earth whereas in the counter-direction it would be increased by just that value. To prove this, they set up an experiment consisting of a stone block floating in a mercury bath, where a monochromatic light beam was split in two beams reflected by mirrors mounted at right angles. Minimum speed variations of any beam should lead to a change in the interference patterns on an observation screen. Rotating the whole experimental arrangement should lead to the expected speed variations. But against the prevailing theory, nothing happened. [1]

It took until 1905 to provide a consistent, working model, and it was Albert Einstein who in his special theory of relativity (STR) postulated that all linear moving systems were equivalent. In Einstein's view, all physical constants were independent of the velocity of the observed system, even the speed of light. With this, ether as a stationary framework became redundant.

Evidences for the validity of STR

Ever since 1905 the Michelson-Morley experiment has been repeated and refined at regular intervals. Since October 2002 this experiment has been conducted with unprecedented accuracy by a work group at Humboldt University Berlin. The team around Achim Peters measures the propagation time of a light beam that is reflected about 100,000 times in an optical resonator. Despite highest precision, the scientists were not able to detect any deviation from STR. [2]

What is more, already in 1963 time dilatation was indirectly proven by the scientists Frisch and Smith in the scope of a muon-counting experiment. Muons are originated at 15 km altitude in the upper atmosphere and move at nearly the speed of light (0.9994c). As their half-life is only 1.5µs, half of them should decay already after 450m. However, this is clearly contradictory to the fact that a large percentage of muons were counted at the ground. This phenomenon of particles surviving on their way down to earth could only be explained by an increased particle lifetime as a result of time dilatation. [3] These results have also been confirmed by the nuclear research center CERN with the aid of artificially created muons within a storage ring. [4] Here, the relativistic half-life of high-speed muons corresponds to the time dilatation formula with a variation of just 1%.

Even more direct is the evidence for time dilatation that the physicists Hafele and Keating supplied in 1971 during measurements with atomic clocks on board of airplanes. In accordance with Einstein's time dilatation formula it turned out that atomic clocks, too, went slower than comparable stationary clocks on ground. [5]

Inconsistencies of STR

Despite the overwhelming burden of proof there still are some inconsistencies within the postulates of STR.

For example, in a computer simulation of a relativistic universe a stationary framework is required to enable calculation of Einstein's twin paradox. In the muon-counting experiment, we chose the earth as a super-ordinate inertial system. If we want to determine the time dilatation between two galaxies that are drifting away from each other we likewise require a superior inertial system, which in the end correspondents to a stationary cosmic framework we might call ether.

In 1965, the actual existence of such a stationary cosmic framework was proven by Arno Penzias and Robert Wilson, who discovered an "echo" of the Big Bang in the form of an extremely homogeneous background noise. In 1976 and 1977 Richard Muller and his team tried to measure this 3k-radiation energy with highest precision. During their experiments on board of a U2 aircraft they detected that the 3k-radiation was red-shifted in direction of the Leo constellation. From this effect the scientists concluded that not only was the earth orbiting the sun with about 30km/s, but also the whole Milky Way raced away from an imaginary center of the universe at a furious speed of nearly 650km/s. [6]

And even Einstein himself mentioned in his lecture delivered at the University of Leiden on May 5th 1920: "Recapitulating: we may say that according to the general theory of relativity space is endowed with physical qualities; in this sense, therefore, ether exists. According to the general theory of relativity, space without ether is unthinkable; for in such space not only would there be no propagation of light, but also no possibility for the existence of standards of measuring rods and clocks, nor therefore any space-time intervals in the physical sense." [7]

Relativistic calculation of the Michelson-Morley experiment

As shown previously, both the STR as well as the ether theory were validated by experiment. But it was not until the attempted relativistic calculation of the Michelson-Morley experiment for computer simulation purposes that – quite unexpectedly - the solution to a one hundred year long scientific quarrel was found.

Initial conditions

For the relativistic calculation of the Michelson-Morley experiment we must distinguish three different perspectives. First, the absolute stationary system (perspective 0). Second, the experiment must be analyzed in parallel to the direction of motion from a stationary observer's point of view (perspective V). Finally, the course of beams propagating at right angle to the velocity has to be calculated also from a stationary observer's point of view (perspective H). Additionally, it must be differentiated between the distance of mirrors (=length l) and the actual distance s that a light beam has to cover compared to a stationary system.

To demonstrate both, validity of STR as well as admissibility of an entirely stationary framework (ether), it is imperative for both models that there are no measurable changes in the interference patterns of the Michelson-Morley experiment. This prerequisite is met if the number of wave maxima N remains constant across the entire distance covered by the beams, irrespective of an inertial system's direction and velocity, and of the chosen perspective. Thus, the following has to apply: $N_0 = N_V = N_H$.

Perspective V: Classical calculation of wave maxima

Michelson and Morley expected that in case of an absolute stationary framework the propagation speed of light would depend on the velocity v of an inertial system, meaning that in the direction of motion a light beam would need longer to cover the distance l:

$$t_{V1} = \frac{l_V}{c - v} \tag{1}$$

By analogy, in the counter-direction of motion, the speed of the inertial system would have to be added to the speed of light, while the propagation time would decrease accordingly:

$$t_{V2} = \frac{l_V}{c + v} \tag{2}$$

For both directions, the light beam will need the time t_V, that can be derived from t_{V1} and t_{V2}:

$$t_V = t_{V1} + t_{V2} = \frac{l_V}{c - v} + \frac{l_V}{c + v} \tag{3}$$

Slightly transformed, this results in the following eq.:

$$t_V = \frac{2l_V}{c} \cdot \frac{1}{1 - \frac{v^2}{c^2}} \tag{4}$$

Striking is the fact that the propagation time of the light beam is multiplied by the quadratic gamma factor in the Lorentz transformation.

For all values of $v > 0$ it can be considered that $t_V > t_0$, where t_0 is the time required by a light beam in a stationary system to cover the distance l in both directions. In this case, the balance of the light beam can be expected to be disturbed, causing a shift of wave maxima that, in turn, will lead to a change in the interference patterns.

Translated into mathematical terms, the number of wave maxima will change over the whole distance. In the stationary system this number can be derived from the frequency f of the monochromatic light beam and its total propagation time t_0.

$$N_0 = f_0 \cdot t_0 \tag{5}$$

For the moving system the increased propagation time t_V has to be considered:

$$N_V = f_V \cdot t_V \tag{6}$$

If, in a next step, the eqs. for t_0 and t_v are inserted in eq. (6) it becomes obvious that there will be more wave maxima in the direction of motion than at right angle to the direction of motion. From this we can expect a change in the interference patterns:

$$N_V = f_V \frac{2l_V}{c} \cdot \frac{1}{1-\frac{v^2}{c^2}} \neq N_0 = f_0 \frac{2l_0}{c} \tag{7}$$

Perspective V: Consideration of length contraction

As shown previously, there is no doubt regarding the validity of the two STR core messages, i.e. time dilatation and length contraction. Let us therefore consider the experiment from a relativistic point of view, taking into account all measured values and applying them to a stationary framework. First, we have to include the length contraction of distance l in the direction of motion. For this, we use the appropriate Lorentz eq.:

$$l_V = l_0 \sqrt{1-\frac{v^2}{c^2}} \tag{8}$$

Here, l_0 corresponds to the original distance and l_V to the new distance which, from a stationary point of view, is shorter. A new eq. results for the total propagation time in the direction of motion:

$$t_V = \frac{2l_V}{c} \cdot \frac{1}{1-\frac{v^2}{c^2}} \tag{9}$$

If we apply the length contraction to eq. (9), we obtain the eq. for the stationary propagation time supplemented by the gamma factor for time dilatation:

$$t_V = \frac{2l_0}{c} \cdot \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{10}$$

Though length contraction and time dilatation have been included in the calculation for the Michelson-Morley experiment, there still is a deviation of the number of wave maxima:

$$N_V = f_V \frac{2l_0}{c} \cdot \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \neq N_0 = f_0 \frac{2l_0}{c} \tag{11}$$

Perspective V: Consideration of time dilatation

When analyzing the above eq. in more detail it turns out that a time-dependent factor has still not been compensated for in accordance with STR principles: the frequency. In a system decelerated by time dilatation the frequencies are affected as well, and so is the frequency of light. However, this blueshift is not equated with the Doppler Effect, but is in fact the subjective view of the moving observer. The faster he moves the stronger time is dilated; constant frequencies are increased in a time-lapse way. For this reason, if we take the frequency to be an inverse function of time, we will also have to apply the inverse gamma factor for time dilatation to this frequency:

$$f_V = f_0\sqrt{1-\frac{v^2}{c^2}} \tag{12}$$

Inserted into the eq. for the number of wave maxima we obtain the following result:

$$N_V = f_V\frac{2l_0}{c}\cdot\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = f_0\sqrt{1-\frac{v^2}{c^2}}\cdot\frac{2l_0}{c}\cdot\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

The two compensation factors can be canceled out and the velocity no longer has an effect on the number of wave maxima. The eq. is identical for the moving as well as the stationary system:

$$N_V = f_0\frac{2l_0}{c} = N_0 \quad \textbf{Q.E.D.} \tag{13}$$

Recapitulating, we can say that regardless of the velocity of an inertial system the number of wave maxima in a moving system will always be the same as in a stationary system.

Perspective H: Transformation at right angle to the direction of motion

Next, we have to examine whether this transformation affects the number of wave maxima N_H at right angle to direction of motion. For in this case, as well, it has to apply that $N_H = N_0$.

As there is no length contraction in the upright direction of motion, we expect the following eq. to be valid:

$$t_H = \frac{2l_0}{c} \tag{14}$$

Here, t_H is the propagation time a light beam needs in the right angle direction. The number of wave maxima N_H can be calculated from

$$N_H = f_H \cdot t_H \tag{15}$$

where f_H is the emission frequency of a monochromatic light source within the moving system. Note that the emission frequency is not the Doppler frequency, but the frequency of a monochromatic light-emitting radiation source. Accordingly, this frequency has to be constant within the whole moving inertial system.
It can be considered as $f_H = f_V$:

$$f_H = f_0\sqrt{1-\frac{v^2}{c^2}} \tag{16}$$

If we incorporate this in eq. (15) we can now actually measure changes in the interference pattern of the light beam propagating at right angle to the direction of motion:

$$N_H = f_0 \frac{2l_0}{c} \cdot \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \quad \neq \quad N_0 = f_0 \frac{2l_0}{c} \tag{17}$$

This result is surprising, as we expected the behavior of the upright light beam to be just like in the stationary system. But when looked at in more detail, this approach turns out to be greatly simplified and thus not valid for all parameters.

First, it is assumed that for all axes within an inertial system time is delayed in the same way. Therefore, time dilatation has to be applied to all directions of motion.

However, if we take a look at the upright light beam we will find that from the point of view of a stationary observer this beam passes through an isosceles triangle, the base of which depends on the relative velocity of the moving system. Despite an unchanged distance between the mirrors l_H the light beam has to bridge a distinctly higher distance s_H. [8]

For this reason, a light beam at right angle to the direction of motion takes just as long to cover the distance between the mirrors as a light beam within a stationary system. Put into mathematical terms this means that time dilatation is also valid uprightly to the direction of motion:

$$t_H = t_0 \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{18}$$

However, the increased distance s_H which the light beam has to cover compensates for the influence of time dilatation:

$$s_H = s_{H1} + s_{H2} = 2l_0 \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{19}$$

If we use these eqs. to calculate the number of wave maxima upright to the direction of motion, there will be a new result:

$$N_H = f_H \cdot t_H = f_0 \sqrt{1-\frac{v^2}{c^2}} \cdot t_0 \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{20}$$

After canceling out the gamma factor we obtain the same number of wave maxima as in a stationary system:

$$N_H = f_0 \cdot t_0 \quad = \quad N_0 \qquad \textbf{Q.E.D.} \tag{21}$$

Conclusion

By applying the Lorentz transformation to the Michelson-Morley experiment it has been shown that the relative velocity of a moving inertial system has no effects on the resulting interference patterns. However, this statement is only valid if both of Einstein’s postulates, length contraction and time dilatation, actually occur.

This paves the way for the introduction of an absolute stationary framework to the classical STR. That would resolve the known inconsistencies within the STR and, at the same time, prove the validity of the central STR postulates, time dilatation and length contraction.

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